Bull. Math. Sci. (2017) 7:201–210
DOI 10.1007/s13373-016-0087-0

Blowing-up solutions for a nonlinear time-fractional system

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Received: 29 April 2016 / Revised: 12 June 2016 / Accepted: 14 June 2016 /
Published online: 21 June 2016

Abstract A nonlinear system with different fractional derivative terms is considered. The existence of positive blowing-up solutions is proved.

Keywords Fractional calculus · Blowing-up solutions · Blowing-up profile

Mathematics Subject Classification Primary 35R11; Secondary 35A01

Communicated by Neil Trudinger.

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1 Introduction

We are concerned with blowing-up solutions of the nonlinear fractional system:

$$\begin{cases} u'(t) - D_t^{\alpha}(u - u(0))(t) = u^p(t)v^q(t), \ t > 0, \\ v'(t) - D_t^{\beta}(v - v(0))(t) = u^r(t)v^s(t), \ t > 0, \\ u(0) = u_0 > 0, \ v(0) = v_0 > 0, \end{cases} \tag{1.1}$$

for $u > 0, v > 0$, where D_t^{σ} for $0 < \sigma < 1$ ($\sigma = \alpha, \beta$) stands for the Riemann-Liouville fractional derivative defined for an integrable function f by $(D_t^{\sigma} f)(t) = \frac{1}{\Gamma(\sigma-1)} \frac{d}{dt} \int_0^t \frac{f(\tau)}{(t-\tau)^{\sigma}} d\sigma$, p, q, r, s are positive real numbers to be fixed later. There are a couple of physical motivations for considering the system (1.1). Firstly, the type of nonlinearities in the system (1.1) appears in the systems describing processes of heat diffusion and combustion in two component continua with nonlinear heat conduction and volumetric release ($u_t - a\Delta u = u^p v^q, \ v_t - b\Delta u = u^r v^s$, the subscript t stands for the time derivative, while Δ stands for the Laplacian operator) [6]. Secondly, as suggested recently [2] one may take $\Delta D_t^{\alpha}(u - u(0))$ and $\Delta D_t^{\beta}(v - v(0))$ instead of Δu and Δv if the process takes place in a porous medium. To simplify the analysis one may start by replacing $\Delta(u - u(0))$ and $\Delta(v - v(0))$ by $(u - u(0))$ and $(v - v(0))$, respectively.

Before, we state our results, let us dwell a while on the existing literature. In [3], Furati and Kirane considered blowing-up solutions to the system

$$\begin{cases} u'(t) + D_t^{\alpha}(u - u(0))(t) = v^q(t), \ t > 0, \\ v'(t) + D_t^{\beta}(v - v(0))(t) = u^r(t), \ t > 0, \\ u(0) = u_0 > 0, \ v(0) = v_0 > 0, \end{cases} \tag{1.2}$$

for $u > 0, v > 0$ and $0 < \alpha, \beta < 1$. Then Kirane and Malik in [4] studied the profile of the blowing-up solutions of system (1.2). The study of the reduced system:

$$\begin{cases} u'(t) = u^p(t)v^q(t), \quad t > 0, \\ v'(t) = u^r(t)v^s(t), \quad t > 0, \\ u(0) = u_0 > 0, \ v(0) = v_0 > 0, \end{cases} \tag{1.3}$$

for $u > 0, \ v > 0$ is well documented in the book [6]; in fact, it admits the first integral

$$\frac{u(t)^{a_1}}{a_1} - \frac{v(t)^{a_2}}{a_2} = C_0 = \frac{u_0^{a_1}}{a_1} - \frac{v_0^{a_2}}{a_2}, \ t > 0, \tag{1.4}$$

where $a_1 = r + 1 - p, \ a_2 = q + 1 - s$. it can then be decoupled in

$$u'(t) = u^p(t)\left(\frac{a_2}{a_1}u^{a_1} - a_2 C_0\right)^{q/a_2}, t > 0, \tag{1.5}$$

$$v'(t) = v^s(t)\left(\frac{a_1}{a_2}v^{a_2} - a_1 C_0\right)^{r/a_1}, \ t > 0. \tag{1.6}$$

From here, the occurrence of finite time blow-up in each component can be derived. If, for example, $a_1 > 0, a_2 > 0$, then u blows-up whenever $p + a_1 q/a_2 > 1$; this inequality is equivalent to the condition $-rq + (p-1)(s-1) > 0$, which is satisfied in this case (since $p < 1+r$, $s < 1+q$). Similarly, it can be checked that if $a_1 > 0$, $a_2 > 0$, the second component v also blows-up in finite time. From the identity (1.4) it follows that the blow-up times of $u(t)$ and $v(t)$ are the same.
A different situation arises when $a_1 a_2 < 0$, for example, if $a_1 > 0$, $a_2 < 0$. In this case $C_0 > 0$, and since $s > 1+q > 1$, v blows-up in finite time: $v(t) \to +\infty$ as $t \to T_0^- < +\infty$. The component $u(t)$ in this case remains bounded: $u(t) \to (a_1 C_0)^{1/a_1}$, $t \to T_0^-$. In the case $a_1 < 0, a_2 < 0$, the constant C_0 can be either sign. For $C_0 = 0$, both components of the solutions of equations (1.5) and (1.6) lead to finite time blow-up. If $C_0 < 0$, then $u(t)$ blows-up, while $v(t)$ remains bounded; if $C_0 > 0$ it is the other way around.

However, such a decoupling and analysis is not directly possible for system (1.1).

2 Results

As argued in [5], (1.1) admits a local solution $(u, v) \in C^1(0, T_{max}) \times C^1(0, T_{max})$. So,

$$D_t^\alpha (u - u_0) = {}^c D_t^\alpha u,$$
$$D_t^\beta (v - v_0) = {}^c D_t^\beta v,$$

where ${}^c D_t^\alpha$ is the so-called Caputo fractional derivative defined, for a differentiable function f, by $({}^c D_t^\alpha f)(t) = \frac{1}{\Gamma(\sigma-1)} \int_0^t \frac{f'(\tau)}{(t-\tau)^\sigma}\, d\sigma$.

For the sequel, we need the following lemma.

Lemma 2.1 *If $\varphi \in C^1(\mathbb{R})$ is increasing and $u' \geq 0$, then, ${}^c D_t^\alpha \varphi(u) \leq \varphi'(u) {}^c D_t^\alpha u$.*

Proof We write

$$\begin{aligned}
{}^c D_t^\alpha \varphi(u) &= \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\frac{d}{ds}\varphi(u(s))}{(t-s)^\alpha} ds \\
&= \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\varphi'(u(s))u'(s)}{(t-s)^\alpha} ds \\
&\leq \frac{\varphi'(u(t))}{\Gamma(1-\alpha)} \int_0^t \frac{u'(s)}{(t-s)^\alpha} ds \\
&\leq \varphi'(u) {}^c D_t^\alpha u.
\end{aligned}$$

Let us recall the Mittag–Leffler function

$$e_{1-\alpha}(t) = E_{\alpha,1}(t^\alpha) = \sum_{n=0}^{\infty} \frac{t^{\alpha n}}{\Gamma(\alpha n + 1)}, \, t > 0,$$

$$e'_{1-\alpha}(t) = \sum_{n=1}^{\infty} \frac{t^{\alpha_n - 1}}{\Gamma(\alpha_n)} > 0, \ t > 0.$$

The solutions of (1.1) satisfies the system of integral equations

$$u(t) = u_0 + \int_0^t e_{1-\alpha}(t-\tau) u^p(\tau) v^q(\tau) d\tau, \tag{2.1}$$

$$v(t) = v_0 + \int_0^t e_{1-\beta}(t-\tau) u^r(\tau) v^s(\tau) d\tau. \tag{2.2}$$

It is clear then that $u(t) \geq u_0$, $v(t) \geq v_0$.

Now, we prove the positivity of u' and v'. □

Lemma 2.2 *We have* $u' > 0$ *and* $v' > 0$.

Proof Let $J^\alpha f(t) = \dfrac{1}{\Gamma(\alpha)} \displaystyle\int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, t > 0, \alpha \in \mathbb{R}^+$.
As ${}^c D^\alpha f(t) = J^{1-\alpha} f'(t)$, we write the first equation of system (1.1) as:

$$u'(t) - J^{1-\alpha} u'(t) = u^p(t) v^q(t),$$

or

$$(I - J^{1-\alpha}) u'(t) = u^p(t) v^q(t),$$

where I is the identity operator; so, formally,

$$\begin{aligned} u'(t) &= (I - J^{1-\alpha})^{-1} u^p(t) v^q(t) \\ &= u^p v^q + \sum_{k=1}^{\infty} \frac{t^{(1-\alpha)k-1}}{\Gamma((1-\alpha)k)} * u^p(t) v^q(t) \\ &= u^p(t) v^q(t) + \int_0^\infty e'_{1-\alpha}(t-\tau) u^p(\tau) v^q(\tau) d\tau \end{aligned}$$

where $*$ denotes convolution. We clearly have $u' > 0$. Similarly, $v' > 0$ can be proved.

As a first consequence, the solution (u, v) of system (1.1) satisfies the system of inequalities

$$\begin{cases} u' \geq u^p v^q, & u(0) = u_0 > 0, \\ v' \geq u^r v^s, & v(0) = v_0 > 0; \end{cases} \tag{2.3}$$

so it is an upper solution of the system

$$\begin{cases} \tilde{u}' = \tilde{u}^p \tilde{v}^q, \ u(0) = u_0 > 0, \\ \tilde{v}' = \tilde{u}^r \tilde{v}^s, \ v(0) = v_0 > 0. \end{cases} \tag{2.4}$$

Our first result is the following blow-up result concerning solutions to system (1.1). □

Theorem 2.3 *We have the two results:*

- *If* $a_1 = r+1-p > 0$, $a_2 = q+1-s > 0$ *and* $p+(r+1-p)q/(q+1-s) > 1$, *then the solution* $(u,\ v)$ *of system* (1.1) *blows-up in finite time.*
- *If* $C_0 = 0$, *then the solution* $(u,\ v)$ *of system* (1.1) *blows-up in finite time.*

Proof The proof is based on the results above concerning system (1.3).

Before we state our next result, let us set $\rho = r/(1-p)$, $\gamma = q/(1-s)$, $\rho' = r/(r-1+p)$ (so $\rho+\rho' = \rho\rho'$), $\gamma' = s/(s-1+q)$ (so $\gamma+\gamma' = \gamma\gamma'$). Our second result is the following theorem. □

Theorem 2.4 *Let* (u, v) *be the solution of* (1.1) *associated to the initial condition* (u_0, v_0). *If* $0 < p < 1,\ 0 < s < 1,\ r > 1-p,\ q > 1-s$, *and if the condition*

$$1 - \frac{1}{\rho\gamma} \le \beta + \frac{\alpha}{\gamma} \quad \text{or} \quad 1 - \frac{1}{\rho\gamma} \le \alpha + \frac{\beta}{\rho}, \tag{2.5}$$

is satisfied, then (u, v) *blows-up in a finite time.*

Proof Multiplying the first equation of (1.1) by $\theta u^{\theta-1}$, we obtain

$$\theta u^{\theta-1} u^p v^q = \theta u^{\theta-1} u' - \theta u^{\theta-1} D_t^\alpha (u-u_0) = \theta u^{\theta-1} u' - \theta u^{\theta-1} ({}^c D_t^\alpha u). \tag{2.6}$$

Using Lemma 2.2 with $\varphi(u) = u^\theta$, we have

$${}^c D_t^\alpha u^\theta \le (\theta u^{\theta-1})({}^c D_t^\alpha u),$$

so,

$$-\theta u^{\theta-1} D_t^\alpha (u-u_0) \le -{}^c D_t^\alpha u^\theta.$$

Whereupon,

$$\theta u^{\theta+p-1} v^q \le (u^\theta)' - {}^c D_t^\alpha u^\theta. \tag{2.7}$$

In a similar manner, multiplying the second equation of (1.1) by $\lambda v^{\lambda-1}$ and using Lemma 2.1, we obtain

$$\lambda u^r v^{s-1+\lambda} \le (v^\lambda)' - {}^c D_t^\beta v^\lambda. \tag{2.8}$$

Now, setting $u^\theta =: U$, $v^\lambda =: V$, the inequalities (2.7) and (2.8) take the form

$$\theta U^{\frac{\theta+p-1}{\theta}} V^{\frac{q}{\lambda}} \le U' - {}^c D_t^\alpha U, \tag{2.9}$$

$$\lambda U^{\frac{r}{\theta}} V^{\frac{s+\lambda-1}{\lambda}} \le V' - {}^c D_t^\beta V. \tag{2.10}$$

Let us choose

$$\begin{cases} \theta + p - 1 = 0 & \Leftrightarrow \quad \theta = 1-p, \\ \lambda + s - 1 = 0 & \Leftrightarrow \quad \lambda = 1-s. \end{cases}$$

Consequently, the system of Eqs. (2.9) and (2.10) can be written

$$\theta V^{\gamma} \leq U' -^{c} D_t^{\alpha} U, \tag{2.11}$$

$$\lambda U^{\rho} \leq V' -^{c} D_t^{\beta} V, \tag{2.12}$$

where $q/\lambda = \gamma, r/\theta = \rho$.
Now we multiply (2.11) by

$$\varphi(t) = \begin{cases} \left(1 - \dfrac{t}{T}\right)^{\sigma}, & t \leq T, \ \sigma >> 1, \\ 0, & t \geq T, \end{cases}$$

and integrate over $(0, T)$; we obtain

$$\theta \int_0^T V^{\gamma} \varphi(\tau) d\tau \leq \int_0^T U'(\tau)\varphi(\tau) d\tau - \int_0^T ({}^c D_t^{\alpha} U(\tau))\varphi(\tau) d\tau. \tag{2.13}$$

First, we compute:

$$\begin{aligned} \int_0^T U'(\tau)\varphi(\tau) d\tau &= U\varphi\Big|_0^T - \int_0^T U(\tau)\varphi'(\tau) d\tau \\ &= -U(0) - \int_0^T U(\tau)\varphi'(\tau) d\tau \\ &\leq -U(0) + \int_0^T U(\tau)|\varphi'(\tau)| d\tau. \end{aligned}$$

Writing

$$\int_0^T U(\tau)|\varphi'(\tau)|\, d\tau = \int_0^T U(\tau)\varphi^{\frac{1}{\rho}}(\tau)\varphi^{-\frac{1}{\rho}}(\tau)|\varphi'(\tau)|\, d\tau,$$

and then using Holder's inequality, we obtain the estimate

$$\int_0^T U(\tau)|\varphi'(\tau)|\, d\tau \leq \left(\int_0^T U^{\rho}(\tau)\varphi(\tau)\, d\tau\right)^{\frac{1}{\rho}} \left(\int_0^T \varphi^{-\frac{\rho'}{\rho}}(\tau)|\varphi'(\tau)|^{\rho'}\, d\tau\right)^{\frac{1}{\rho'}}, \tag{2.14}$$

where $\rho' + \rho = \rho\rho'$.
Alike, we obtain the estimate

$$\begin{aligned} \int_0^T ({}^c D_\tau^{\alpha} U(\tau))\varphi(\tau)\, d\tau &= \int_0^T U(\tau)({}^c D_{\tau|T}^{\alpha}\varphi(\tau))\, d\tau \\ &\leq \left(\int_0^T U^{\rho}(\tau)\varphi(\tau)\, d\tau\right)^{\frac{1}{\rho}} \\ &\quad \times \left(\int_0^T \varphi^{-\frac{\rho'}{\rho}}(\tau)|{}^c D_{\tau|T}^{\alpha}\varphi(\tau)|^{\rho'}\, d\tau\right)^{\frac{1}{\rho'}}. \end{aligned} \tag{2.15}$$

Setting

$$A(\varphi,\rho)=\left(\int_0^T \varphi^{-\frac{\rho'}{\rho}}(\tau)|\varphi'(\tau)|^{\rho'}d\tau\right)^{\frac{1}{\rho'}},$$

and

$$B(\varphi,\rho)=\left(\int_0^T \varphi^{-\frac{\rho'}{\rho}}(\tau)|^c D^\alpha_{\tau|T}\varphi(\tau)|^{\rho'}d\tau\right)^{\frac{1}{\rho'}},$$

in (2.14) and (2.15) and inserting the resulting expressions in (2.13), we get

$$\theta\int_0^T V^\gamma\varphi\,d\tau+U(0)\le\left(\int_0^T U^\rho\varphi d\tau\right)^{\frac{1}{\rho}}A(\varphi,\rho)+\left(\int_0^T U^\rho\varphi\,d\tau\right)^{\frac{1}{\rho}}B(\varphi,\rho).$$

Similarly, we obtain

$$\lambda\int_0^T U^\rho\varphi+V(0)\le\left(\int_0^T V^\gamma\varphi\,d\tau\right)^{\frac{1}{\gamma}}A(\varphi,\gamma)+\left(\int_0^T V^\gamma\varphi\,d\tau\right)^{\frac{1}{\gamma}}B(\varphi,\gamma).$$

Setting $I=\int_0^T U^\rho\varphi\,d\tau$ and $J=\int_0^T V^\gamma\varphi\,d\tau$, we have

$$\theta J+U(0)\le I^{\frac{1}{\rho}}\Big(A(\varphi,\rho)+B_\alpha(\varphi,\rho)\Big),\tag{2.16}$$

$$\lambda I+V(0)\le J^{\frac{1}{\gamma}}\Big(A(\varphi,\gamma)+B_\beta(\varphi,\gamma)\Big).\tag{2.17}$$

Using $I\le CJ^{\frac{1}{\gamma}}\Big(A(\varphi,\gamma)+B_\beta(\varphi,\gamma)\Big)$ in (2.16), we obtain

$$J+U(0)\le CJ^{\frac{1}{\rho\gamma}}\Big(A(\varphi,\gamma)+B_\beta(\varphi,\gamma)\Big)^{\frac{1}{\rho}}\Big(A(\varphi,\rho)+B_\alpha(\varphi,\rho)\Big),$$

which implies

$$J^{1-\frac{1}{\rho\gamma}}\le C\Big(A(\varphi,\gamma)+B_\beta(\varphi,\gamma)\Big)^{\frac{1}{\rho}}\Big(A(\varphi,\rho)+B_\alpha(\varphi,\rho)\Big).\tag{2.18}$$

Similarly, we have $J\le CI^{\frac{1}{\rho}}\Big(A(\varphi,\rho)+B_\alpha(\varphi,\rho)\Big)$, and

$$I+V(0)\le CI^{\frac{1}{\rho\gamma}}\Big(A(\varphi,\rho)+B_\beta(\varphi,\rho)\Big)^{\frac{1}{\gamma}}\Big(A(\varphi,\gamma)+B_\alpha(\varphi,\gamma)\Big).$$

Thus

$$I^{1-\frac{1}{\rho\gamma}} \leq C\Big(A(\varphi,\rho)+B_\beta(\varphi,\rho)\Big)^{\frac{1}{\gamma}}\Big(A(\varphi,\gamma)+B_\alpha(\varphi,\gamma)\Big). \tag{2.19}$$

As $\int_0^T \varphi^{\vartheta}|^c D^{\alpha}_{t|T}\varphi|^{\widetilde{\rho}}dt = C_{\widetilde{\rho},\alpha}T^{1-\alpha\widetilde{\rho}}$, where ϑ is any exponent, we have:

$$B(\varphi,\rho)=CT^{\frac{1-\alpha\rho'}{\rho'}}, \quad A(\varphi,\rho)=CT^{\frac{1-\rho'}{\rho'}}. \tag{2.20}$$

Inserting (2.20) into (2.19), we obtain the estimate

$$\begin{aligned} I^{1-\frac{1}{\rho\gamma}} &\leq C\left(T^{\frac{1-\rho'}{\rho'}}+T^{\frac{1-\alpha\rho'}{\rho'}}\right)^{\frac{1}{\gamma}}\left(T^{\frac{1-\gamma'}{\gamma'}}+T^{\frac{1-\beta\gamma'}{\gamma'}}\right)\\ &\leq CT^{\frac{1-\alpha\rho'}{\gamma\rho'}}T^{\frac{1-\beta\gamma'}{\gamma'}}, \quad for\ \ T>>1\\ &\leq CT^{-\left(\frac{1}{\rho\gamma}+\frac{\alpha}{\gamma}\right)+1-\beta}. \end{aligned} \tag{2.21}$$

Assume by contradiction that the solution is global and bounded. Then, by letting $T\to+\infty$ in (2.21), we obtain a contradiction if

$$-\left(\frac{\alpha}{\gamma}+\frac{1}{\rho\gamma}\right)+1-\beta<0 \quad \Leftrightarrow \quad 1-\frac{1}{\rho\gamma}\leq\beta+\frac{\alpha}{\gamma}.$$

Alike, similar estimates along J leads to a contradiction if

$$1-\frac{1}{\rho\gamma}\leq\alpha+\frac{\beta}{\rho}.$$

The case for $1-\frac{1}{\rho\gamma}=\beta+\frac{\alpha}{\gamma}$ or $1-\frac{1}{\rho\gamma}=\alpha+\frac{\beta}{\rho}$ can be treated as in [4]. □

3 Estimate of the blow-up time and profile of the solution

We consider only solutions under conditions of Theorem 2.4.

Without loss of generality, we assume that $\theta>\lambda\Leftrightarrow s>p$. Then we have:

$$\begin{aligned} U' &\geq \lambda V^{\sigma},\ \sigma=\frac{q}{\lambda}>1,\\ V' &\geq \lambda U^{\delta},\ \delta\frac{r}{\rho}>1; \end{aligned}$$

so $U\geq\tilde{U}$ and $V\geq\tilde{V}$ where

$$\tilde{U}'(\tau)=\tilde{V}^{\sigma}(\tau), \quad \tilde{U}_0=U_0, \tag{3.1}$$
$$\tilde{V}'(\tau)=\tilde{U}^{\delta}(\tau) \quad \tilde{V}_0=V_0. \tag{3.2}$$

Solutions of (3.1) and (3.2) are explicitly given by

$$\widetilde{U}(\tau) = \widetilde{C_1}(\widetilde{T_{max}} - \tau)^{-\frac{1+\delta}{\sigma\delta-1}}$$

and

$$\widetilde{V}(\tau) = \widetilde{C_2}(\widetilde{T_{max}} - \tau)^{-\frac{1+\sigma}{\sigma\delta-1}},$$

where

$$\widetilde{C_1} = \left(\frac{(1+\sigma)(1+\delta)^{\sigma}}{(\sigma\delta-1)^{\delta+1}}\right)^{\frac{1}{\sigma\delta-1}},$$
$$\widetilde{C_2} = \left(\frac{(1+\delta)(1+\sigma)^{\delta}}{(\sigma\delta-1)^{\sigma+1}}\right)^{\frac{1}{\sigma\delta-1}}$$

and

$$\widetilde{T_{max}} = \left(\frac{U_0\,\widetilde{C_2}}{V_0\,\widetilde{C_2}}\right)^{\frac{1}{\delta-\sigma}}.$$

Whereupon,

$$U(t) \geq C_1(T_{max} - t)^{-\frac{1+\delta}{\sigma\delta-1}}$$
$$V(t) \geq C_2(T_{max} - t)^{-\frac{1+\sigma}{\sigma\delta-1}},$$

where

$$C_1 = \widetilde{C_1}\lambda^{-\frac{1+\sigma}{\sigma\delta-1}}, \qquad C_2 = \widetilde{C_2}\lambda^{-\frac{1+\delta}{\sigma\delta-1}},$$

and

$$T_{max} = \widetilde{T_{max}}/\lambda.$$

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